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Non-systematic errors of monthly oceanic rainfall derived from passive microwave radiometry

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Abstract. We compute the non-systematic errors of monthly oceanic rainfall over 5^0 latitude by 5^0 longitude boxes derived from data taken by the Special Sensor Microwave Imager (SSM/I) on board the Defense Meteorological Satellite Program (DMSP) satellites and TRMM Microwave Imager (TMI). The mean rain rates between 1998 and 1999 are calculated to be 3.0, 2.85, 2.89 mm day^{-1} and the non-systematic errors 22.2%, 22.4% and 19.7% for SSM/I onboard the DMSP F13, F14 and TMI, respectively. The non-systematic error for the TMI is smaller than that for one SSM/I at the low rain rates but is comparable at rain rates higher than about 5mm day^{-1}. The TRMM objective of 1mm day^{-1} for non-systematic error is met by TMI for rain rates up to 5-6mm day^{-1}. Improved space-borne sampling strategy, such as that proposed for the Global Precipitation Mission (GPM), is needed to achieve errors better than 10% at the higher rain rates.

1. Introduction

A major objective of the Tropical Rainfall Measuring Mission (TRMM) is to produce time series of monthly rainfall over areas of 5^0 x 5^0 latitude/longitude boxes with an accuracy of 1 mm day^{-1} at the low rain rates and 10% at the high rain rates (Simpson et al., 1988). While rainfall "ground truth" data are relatively hard to find, we can generally distinguish three types of errors in the estimation of space-time rainfall from satellite measurements: systematic, random and sampling (Wilheit, 1988; Chang and Chiu, 1999). Errors in the algorithm assumptions and sensor calibration are the main contributors to systematic errors. Non-systematic errors consist of random and sampling errors. Random error refers to error due to noise in the sensor measurements, such as stray thermal emission from various microwave components. Sampling error arises because only snap shots of the rain fields are measured, and the total space/time rainfall is estimated by interpolation or extrapolation. The sampling error is determined by the orbit of the satellite, the swath width of the instrument, and the space/time characteristics of the rainfall fields which one intends to measure in the first place. To the extent that bias errors can be calibrated to some "ground truth" rainfall, sampling error is by far the most important error source for monthly mean estimates (Bell et al., 1990).

The sampling error of space/time rainfall has been examined by a number of investigators. A recent summary is given by Bell and Kundu (2000). Bell and Kundu (1996) suggest that the rms error varies approximately as the inverse square root of the mean rain rate and sample volume, viz

$$s/\langle R\rangle \sim (\langle R\rangle\, A\, S)^{-1/2}\,. \qquad (1)$$

This equation predicts that sampling error decreases with grid box size A and satellite visit S. S is the sum of the box fractions observed by the satellite during the month. Based on the rain rate statistics derived from the GATE, it was estimated that over an area of 500 km x 500 km, the TRMM goal of providing monthly average rain over a 5^0 x 5^0 latitude and longitude area with an accuracy of 1 mm day^{-1} or 10% in heavy rain cases can be achieved (Kedem et al., 1990; Bell et al., 1990; Simpson, 1988).

Monthly oceanic rainfall maps have been produced from Special Sensor Microwave Imager (SSM/I) data taken on board the Defense Meteorological Satellite Program (DMSP) satellites since 1987 (Wilheit et al., 1991). With the launch of the TRMM in November 1997, data from the TRMM Microwave Imager (TMI) becomes available as another independent oceanic rainfall estimates.

The number of TMI pixels within a 5x5 degree box is larger than that of the SSM/I due to its fine spatial resolution. Typically, the average sample number for TMI is about 10^5 per month per grid box as compared to 16,000 for SSM/I. However, the swath width for TMI (760km) is less than that of SSM/I (1400 km). In this study, we quantify the random error as we compare the different combinations of SSM/I and TMI rain estimates.

The non-systematic error of SSM/I oceanic monthly rainfall have been examined by Chang and Chiu (1999). Their error estimation scheme is based on the existence of pairs of independent estimates of the monthly mean. This paper extends their work to the TMI data and presents the total oceanic rainfall (between the 40^0 latitude bands) and the non-systematic error for the period of January 1998 - December 1999. The data are described in Section 2. The technique for error estimation is described in Section 3. Section 4 presents the error estimates from SSM/I and TMI. Section 5 summarizes and discusses the results.

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2. Data

The DMSP satellites are nominally in a sun synchronized orbit with an equatorial crossing time in the morning and afternoon which are 12 hours apart. The SSM/I microwave brightness temperature (T_B) histograms of a combination channel are computed for the AM and PM data, denoted respectively H(AM) and H(PM). Over the course of a month, the rain rate distribution is assumed to follow a lognormal distribution. The parameters of the rain rate distribution are solved iteratively by matching the moments of the T_B histograms to that computed from the rain rate distributions. The goodness of fit is determined by a chi-square statistics. If the Chi-square statistics exceed a maximum threshold, the monthly rainfall estimates are replaced by simple averaging. Based on AM and PM T_B histograms, monthly estimates for the AM and PM data are calculated, viz.

$$RR(AM) = f(h(AM)) \text{ and } RR(PM) = f(h(PM)) \qquad (2)$$

where f represents the nonlinear process by which the monthly rainfall based on the AM data, and RR(AM), the rain rate based on the AM data, are calculated from the histogram of h(AM). The monthly mean rain is the average of the AM and PM estimates RR(mean) = ½ [RR(AM) + RR(PM)]. The TRMM satellite orbit is non-sun-synchronized, and only one monthly TB histogram hence one monthly mean is estimated.

The derived rain-rate indices are then multiplied by a correction factor to account for the beam-filling bias (Wilheit et al., 1991; Chiu et al., 1990). The beam-filling correction factor has been modified to vary as a linear function of the freezing height (FL in km), viz., beam filling correction = 1 + .088*FL as suggested by Wang (1995). For TMI the beam filling correction factor equals to 1 + 0.062*FL.

3. Error Estimation Technique

Following Chang et al. (1993), let x_1 and x_2 represent the pair of independent estimates of the monthly mean, viz.:

$$x_1 = \langle x_1 \rangle + e_1 \qquad (3)$$

$$x_2 = \langle x_2 \rangle + e_2, \qquad (4)$$

where the ensemble averaging, < > , is taken over the appropriate rain rate categories, and e_1 and e_2 are the error associated with the independent estimates. Assuming that the error are unbiased with uncorrelated errors

$$\langle e_i \rangle = 0, \qquad \text{for } i = 1, 2; \qquad (5)$$

$$\langle e_1 e_2 \rangle = 0 \qquad (6)$$

we get the error as

$$\langle e_1^2 \rangle + \langle e_2^2 \rangle = \{\langle (x_1 - x_2)^2 \rangle - (\langle x_1 \rangle - \langle x_2 \rangle)^2\} \qquad (7)$$

In our previous work (Chang and Chiu, 1999), only F13 and F14 rain rates are compared. Since the technique relies on the comparison of data sets, we have to solve one equation with two unknowns, and the assumption that the rms errors for F13 and F14 are identical was made, i.e. $\langle e_{F13}^2 \rangle = \langle e_{F14}^2 \rangle = \langle e^2 \rangle$. The error is estimated as

$$\langle e^2 \rangle = ½ \{\langle (R_1 - R_2)^2 \rangle - (\langle R_1 \rangle - \langle R_2 \rangle)^2\} \qquad (8)$$

In this study, however, there are three rain rate estimates, and hence there are three equations with three unknowns. This allows us to calculate the errors individually.

$$\langle e_i \rangle^2 = ½ S - V_{j,k} \qquad (9)$$

where i, j, k, are not equal and denote the TMI, F13 and F14 data. S and $V_{j,k}$ are defined as follows

$$S = \langle (R_{TMI} - R_{F13})^2 \rangle + \langle (R_{F13} - R_{F14})^2 \rangle + \langle (R_{F14} - R_{TMI})^2 \rangle \qquad (10)$$

$$V_{j,k} = \{\langle (R_j - R_k)^2 \rangle - (\langle R_j \rangle - \langle R_k \rangle)^2\} \qquad (11)$$

We partitioned the monthly rain rates into 1.5 mm/day rain rate categories and computed the rms error using (7) for these rain rate categories and for all the rain rate categories (hereafter referred to as the total rain rate).

4. Results

We first calculate the errors associated with the F13 and F14 SSM/I data sets by assuming that the non-systematic errors are identical for F13 and F14, viz. $\langle e_1^2 \rangle = \langle e_2^2 \rangle$ as in (8). The calculation is based on two years (1998-1999) of data. When the data are fitted to the form

$$s/\langle R \rangle = a \langle R \rangle^b \qquad (12)$$

where s is the rms error and <R> the mean rain rate in mm day^{-1}, and a and b are fitting parameters, we find (a,b) = (37.4%, -0.369). These coefficients can be compared with those derived from three earlier years (1992-1994) of data (Chang and Chiu, 1999) which showed (a, b)=(42%, -0.366). Note that b is relatively unchanged whereas a is larger for 1992-1994 than 1998-1999, and hence the total error.

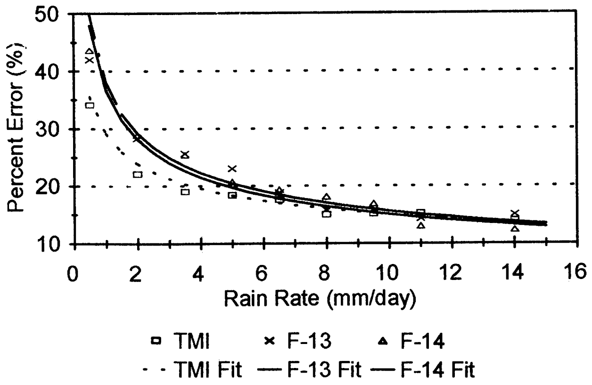

Figure 1. Percent error as a function of the mean rain rate over 5 x 5 degree grid for (a) TMI (square), (b) F13 (cross), and (c) F14 (triangle), for January 1998 to December 1999

Table 1. Non-systematic error of TMI, F-13 and F-14 SSM/I. N is the average number of grid boxes in a rain rate category, <R> the average rain rate and s is the rms error.

TMI	0-1.5	1.5-3	3-4.5	4.5-6	6-7.5	7.5-9	9-10.5	10.5-12	>12
N	314.0	172.9	109.6	67.6	43.1	25.5	11.1	9.5	9.3
<R>	0.678	2.201	3.678	5.246	6.693	8.188	9.767	11.103	14.12
S	0.231	0.486	0.700	0.964	1.176	1.231	1.470	1.666	1.967
S/<R>	0.341	0.221	0.190	0.183	0.176	0.150	0.150	0.152	0.139

TMI average error = 19.7%. Best power law fit: $s/\langle R\rangle = 0.291 \langle R\rangle^{-0.287}$

F-13	0-1.5	1.5-3	3-4.5	4.5-6	6-7.5	7.5-9	9-10.5	10.5-12	>12
N	291.7	139.3	94.7	62.5	43.1	32.0	19.4	10.4	17.9
<R>	0.568	2.219	3.712	5.178	6.693	8.218	9.707	11.14	13.84
S	0.239	0.627	0.950	1.191	1.254	1.295	1.537	1.590	2.075
S/<R>	0.420	0.283	0.256	0.230	0.187	0.157	0.158	0.142	0.150

F-13 average error = 22.2%. Best power law fit: $s/\langle R\rangle = 0.356 \langle R\rangle^{-0.356}$

F-14	0-1.5	1.5-3	3-4.5	4.5-6	6-7.5	7.5-9	9-10.5	10.5-12	>12
N	298.3	139.4	96.0	58.5	43.9	28.1	15.0	10.7	15.6
<R>	0.562	2.189	3.698	5.186	6.687	8.182	9.629	11.161	14.34
S	0.245	0.633	0.938	1.074	1.287	1.478	1.626	1.446	1.755
S/<R>	0.436	0.289	0.254	0.207	0.193	0.187	0.169	0.130	0.122

F-14 average error = 22.4%. Best power law fit: $s/\langle R\rangle = 0.381 \langle R\rangle^{-0.387}$

Note: The number of grid boxes in each category N (for TMI as example) is determined from the following formula: N(TMI) = n(TMI, F-13) + n(TMI, F-14) - n(F-13, F-14), where $n(x_1, x_2)$ is the number used in computing the square quantities in Eq. (7).

Next, the assumption that the errors are identical is relaxed and the errors are calculated according to (9). Table 1 shows the number of samples (N), the mean rain rate <R> and the percent error (s/<R>) for monthly means derived from TMI, F13 and F14 data. Fig. 1 shows the results and the best power law fits for F13, F14, and TMI data. The results of power law fits of the data to (12) are also included. The error structure for SSM/I F13 and F14 is very similar, and hence give credence to our earlier estimates based on the equality assumption.

The total errors are 22.2%, 22.4% and 19.7% for F13, F14 and TMI respectively. The parameters (a, b) are (.356, -.356) and (.381, -.387), respectively for F13 and F14. These numbers are very close to those derived by assuming the F13 and F14 errors are equal. For TMI, (a, b) = (.295, –0.350). The TMI errors are smaller than either of the F13 and F14 at low rain rate, but become comparable at the higher rain rate (>5-7 mm day^{-1}). At higher rain rates, the estimates may not be stable due to the small sample size.

In order to further examine the dependency on sample number, we divide the data into two regions. Region (1), 20°N to 20°S, contains the major rain belts, i.e. the Intertropical Convergence zone. Region (2), 20°N to 40°N and 20°S to 40°S, encompasses the subtropical highs and hence most of the oceanic dry regions. In Region (1) each of the rain rate categories have at least over 8 samples. The total errors, and a and b are (17.3%, .261, -.267), (17.7%, .313, -.352), (18.3%, .327, -.364) for TMI, F13, and F14, respectively. For Region (2), the total error and parameters are (20.8%, .288, -.272), (23.9%, .311, -.336), and (28.3%, .389, -290), respectively for TMI, F13, and F14. The lower percent error in Region (1) is consistent with the fact that percent error decreases with rain rate (Fig. 1). The parameter b is relatively constant for each sensor, except for F14 in Region (2) which may be due to the absence of high rain samples.

Results from two years (1998 and 1999) of SSM/I data are almost identical to those derived from July 1987 to December 1999. The twelve years average total rain rate and s.d. are 3.0 mm day^{-1} and 0.16 mm day^{-1} for SSM/I. The two years average total rain rate (s.d.) are 3.0 mm day^{-1} (0.19 mm day^{-1}), 2.85 mm day^{-1} (0.19 mm day^{-1}), and 2.89 mm day^{-1} (0.15 mm day^{-1}) for F13, F14, and TMI respectively.

The parameter b is remarkably constant between 1992-1994 and 1998-1999. However, a, and hence the total error are larger for 1992-1994. This may be partially attributed to the large number of missing orbits in 1992-1994. During

1992-1994, the average number of samples for some months is less than 6000, whereas in the later months, the sample number is around 8000 (Chang and Chiu, 1999, Fig. 3). Assuming a dependence on the inverse square root of the sample number, the difference in a (42% for 1992-1994 and 35% 1998-1999) can be reconciled.

5. Summary and Discussion

The production of a long-term global rainfall time series is one of objectives of the Global Precipitation Climatology Project (GPCP) and NASA's Earth Observation System (EOS). The quality of the long-term data set is critical to the utilization of the monthly rainfall products. Our analysis showed the differences between F13, F14, and TMI estimates are less than 5%, suggesting that the microwave derived monthly rain estimates are fairly consistent.

This study shows TMI nonsystematic error in the range of 15% for the high rain rate categories (> 7.5 mm day^{-1}) and about 35% for the low rain rate (1 mm day^{-1}). At rain rates less than 5 mm day^{-1} rain rate error is less than 1 mm day^{-1}. However, at the high rain rate regime (> 5.0 mm day^{-1}) currently derived non-systematic errors are larger than 10%. Assuming that the non-systematic error of F13 and F14 are independent, the combination of F13 and F14 (error reduced by square root of 2) is potentially capable of archiving the 10% criteria. This is not as good as the 10% as predicted from GATE and TOGA/COARE data or the TRMM requirement. The planned Global Precipitation Mission (GPM) advocates for passive microwave sensors on multiple (eight) satellites. This sampling configuration will provide coverage roughly every 3 hours, thus reducing the sampling error to better than ~10% as required.

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