JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 95, NO. D3, PAGES 2217–2227, FEBRUARY 28, 1990

Estimating the Exceedance Probability of Rain Rate by Logistic Regression

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Recent studies have shown that the fraction of an area with rain intensity above a fixed threshold is highly correlated with the area-averaged rain rate. To estimate the fractional rainy area, a logistic regression model, which estimates the conditional probability that rain rate over an area exceeds a fixed threshold given the values of related covariates, is developed. The problem of dependency in the data in the estimation procedure is bypassed by the method of partial likelihood. Analyses of simulated scanning multichannel microwave radiometer and observed electrically scanning microwave radiometer data during the Global Atlantic Tropical Experiment period show that the use of logistic regression in pixel classification is superior to multiple regression in predicting whether rain rate at each pixel exceeds a given threshold, even in the presence of noisy data. The potential of the logistic regression technique in satellite rain rate estimation is discussed.

1. INTRODUCTION

There is a growing awareness in the international scientific community that accurate measurement of rainfall is essential to the progress in understanding the working of the Earth-atmosphere-hydrosphere system. The Tropical Rainfall Measuring Mission (TRMM), initiated by NASA and now being studied jointly by the United States and Japan, is devoted to the systematic study of tropical rainfall and its influence on global climate [*Simpson*, 1988]. France has also initiated the Tropical System Energy Budget (BEST) experiment, dedicated to the measurement of horizontal water fluxes and precipitation in the tropics [*Centre National d'Etudes Spatiales*, 1988].

The estimation of rainfall from satellite measurements poses a great challenge to meteorologists. Rainfall retrieval from satellite measurements is based on covariate information such as radiometric measurements or other physical quantities derived from them. Because of the radiative properties of raindrops, physical retrievals from microwave measurements have been considered the most fruitful avenue of attack. Microwave radiation interacts directly with hydrometeors, and hence the emitted microwave radiance can be related to rainfall rate through a cloud model. *Wilheit et al.* [1977], in a pioneering work, calculated the microwave radiance as a function of rain rate, given the observed profiles of cloud and water vapor, and assuming a uniform field of view (FOV) of the microwave sensor. The typical spatial scale of rain is in general smaller than that of the FOV of microwave sensors, and an error is introduced in estimating rain rates from the brightness temperature of the FOV. This error, the so-called "beam filling" error associated with inhomogeneity within the FOV, has been examined from Global Atlantic Tropical Experiment (GATE) data and from models of rainfall by *Chiu et al.* [1989]. The effect of finite clouds in the FOV on the radiative properties has been addressed by *Kummerow and Weinman* [1988]. The problem with the physical approach is that the rain rate/microwave temperature relation is derived from a model of the atmosphere which contains free parameters, such as the height of the rain column and ice particle size distribution. These parameters have to be calibrated. The use of multiple frequency and dual polarization observations holds great potential to the solution of this problem.

Another approach, the use of statistical techniques, relies on empirical relations between rain rate and cloud properties, such as cloud top temperature and cloud fraction. (See *Barrett and Martin* [1981] for a thorough review.) *Whitney and Herman* [1979] used parameters that are available operationally such as IR temperature, 200- and 850-mbar winds, surface dew points, terrain heights, and parameters derived from them in a screening regression of areal rainfall. The variance explained using a six-term model is only about 70%.

The relationship between high clouds and rainfall has led to the use of a cloud index in estimating areal rainfall [*Arkin*, 1979]. The cloud index is the fractional high cloud area and can be estimated from infrared measurements taken by sensors on the operational NOAA satellites. *Lovejoy and Austin* [1979] included the visible channels to delineate the highly reflective nonprecipitating clouds. Their method has been adopted for operational precipitation forecast in Canada.

A strong linear relationship exists between the instantaneous rain area and areal average rainfall [*Lopez et al.*, 1983; *Chiu*, 1988*a*, *b*; *Rosenfeld et al.*, this issue]. If we consider continuous sampling in space and time, the rain area is multiplied by time and can be interpreted as an "area time integral," which has found application in radar meteorology in estimating rainfall [*Lopez et al.*, 1983; *Doneaud et al.*, 1984]. *Rosenfeld et al.* [this issue] incorporated cloud height information in the linear relation and proposed the height

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Paper number 89JD01654.
0148-0227/90/89JD-01654$05.00

area rainfall threshold (HART) technique for rainfall retrieval in TRMM.

The theoretical basis of this linear relation has been discussed by *Atlas et al.* [this issue] and *Kedem et al.* [1986]. *Kedem et al.* [1986] showed that this linear relationship is a manifestation of the law of large numbers. Let R represent rain rate at a given point in time and space; then the expected value of R, $E(R)$, is given by

$$E(R) = q_\tau P(R > \tau)$$

where τ is a fixed threshold level, $q_\tau = E(R|R > 0)/P(R > \tau|R > 0)$ and depends only on τ and the continuous part of the rain rate distribution. Here $P(x)$ denotes the probability of x. When rain rate is homogeneous in time and space, $E(R)$ can be approximated by the area average rain rate, and $P(R > \tau)$ can be approximated by the fractional area of rain rate exceeding τ.

This argument can be extended to other measurements such as radiances from remote sensors. For example, if S is another instantaneous measurement obtained at a point in space and time, then

$$E(R) = \alpha_\tau P(S > S_\tau)$$

where S_τ is a threshold in S which corresponds to τ in rain rate [*Kedem et al.*, 1986]. Hence we might expect a high correlation between the area average rain rate and the fractional area for which S exceeds S_τ. In the case of Arkin's cloud index technique, S is the infrared radiance.

This linear relationship between areal average rain rate and rain area will prove to be extremely useful in rainfall estimation from space. The beauty of using this linear relation is that once the satellite pixels are classified as above or below a certain rain rate threshold, the areal average rainfall rate can be inferred.

A logistic regression model, which calculates the probability that the areal average rain rate over an FOV of a satellite sensor is above a fixed threshold given covariate measurements, was proposed by *Chiu and Kedem* [1986]. Because of the dependency on the data, the method of partial likelihood is introduced in this paper. It should be noted that measurements of rain rate from remote sensors are not measured directly and that uncertainties exist as to the precise relationship between these variables and rain rate. In this respect, logistic regression is a valuable tool. It relates probabilities of a rainfall event to covariates that govern, or at least influence, the occurrence of the event.

Under some conditions, the probability that rain rate exceeds a certain level has the logistic form. This form allows the formation of a partial likelihood from the time series of rain rate. This in turn justifies inference based on large sample theory.

To examine the applicability of the logistic model in rain rate estimation, the model has to be tested on data. The lack of concomitant rain rate and satellite observations over the oceans has prompted the use of simulated data. The use of simulated data is useful in setting a baseline for comparison with real observations. The measurements taken by the scanning multichannel microwave radiometer (SMMR) on board the Nimbus 7 satellite are simulated using rain rate observations taken during GATE. In this respect, we are also investigating the possibility of estimating rain rate using SMMR, as it would give us an historic time series of rain rate measurements. Coincident and colocated rain rate and microwave measurements taken by the electrically scanning microwave radiometer (ESMR) on board Nimbus 5 (ESMR 5) during the GATE period was assembled by *Short* [1988], who has kindly provided us with the data for analysis.

The logistic model and the partial likelihood method are introduced in section 2. The simulation of SMMR data is described in section 3, and section 4 details results from analysis of simulated SMMR data. Section 5 is devoted to the analysis of ESMR 5 measurements during the GATE, and section 6 presents some concluding remarks.

2. LOGISTIC REGRESSION AND PARTIAL LIKELIHOOD

2.1. *Logistic Regression*

The logistic model is useful in determining the relationship between the distribution of a random variable and a set of covariates. It has been applied in various forms in reliability testing and the analysis of survival data. A detailed treatment is given by *Cox* [1970]. Let R be the rain rate in space and time and $\mathbf{z} = (z_1, z_2, \cdots, z_k)$ be the set of covariates. The logistic model is of the form

$$P(R > r) = [1 + \exp(-\boldsymbol{\beta}'\mathbf{z})]^{-1} \quad (1)$$

where $\boldsymbol{\beta}$ is the set of regression coefficients. A major difference between logistic regression and linear regression is that the latter technique maximizes the variance explained, while in logistic regression, a likelihood function, or probability of an event, is maximized. A derivation of the logistic form from the principle of maximum entropy appears in the appendix.

2.2. *Partial Likelihood*

In the theory of statistical inference, the likelihood principle says that inference from observed data should be based on the likelihood function. However, when dealing with dependent data, the likelihood function is usually unknown or difficult to model. Recent studies show that the assumption of strict knowledge of the likelihood function can be relaxed [*Cox*, 1975]. Satisfactory inference can be obtained from partial likelihoods. The method of partial likelihood allows for dependence of the data and the incorporation of covariate information.

The precise definition of partial likelihood is as follows. Consider the data X_t, $t = 1, 2, \cdots$, possibly arising from a nonstationary process, together with the related auxiliary vector process $\mathbf{Z}_t$. Let H_t denote the history of the joint process $(X_s, \mathbf{Z}_s)$ up to time t. Clearly, the information in H_{t_2} contains that in H_{t_1} for $t_1 < t_2$, so that

$$H_1 \in H_2 \in, \cdots, \in H_s, \cdots \quad (2)$$

Whenever constructing a partial likelihood, it must be defined relative to such a nested sequence of histories. Let $f_t(\cdot, \boldsymbol{\beta})$ denote the conditional density (with respect to some measure μ) of X_t given H_{t-1} parameterized by the vector of parameters $\boldsymbol{\beta}$. The partial likelihood $\mathrm{PL}(\boldsymbol{\beta})$ is defined by the product

$$\text{PL}(\boldsymbol{\beta}) \equiv \prod_{t=1}^{T} f_t(X_t, \boldsymbol{\beta}) \quad (3)$$

When H_t represents the history of X_s, $1 \leq s \leq t$, without recourse to the covariate information $\{\mathbf{Z}_t\}$, (3) is simply the likelihood function itself. The vector $\hat{\boldsymbol{\beta}}$ that maximizes PL($\boldsymbol{\beta}$) is called the maximum partial likelihood estimator (MPLE).

The large sample theory based on (3) parallels that of the usual maximum likelihood theory and has been developed in recent years by several authors, including *Wong* [1986] and E. Slud (unpublished manuscript, 1990). The key element that allows this generalization is that PL($\boldsymbol{\beta}^1$)/PL($\boldsymbol{\beta}$) is a Martingale with respect to the nested sequence in (2). (Recall that $\{W_t\}$ is Martingale with respect to (2) if $E(w_t | H_{t-1}) = W_{t-1}$ [see *Karlin and Taylor*, 1975, p. 306].)

We combine (1) and (3) to arrive at a useful logistic regression model. Let $\{R_t\}$ be a time series of rain rates averaged over an area. By this we mean the instantaneous rain intensities averaged over a given area at times $t = 1, 2, 3, \cdots$. Define the binary time series $\{X_t\}$ by

$$X_t = 1 \qquad R_t \geq r$$

$$X_t = 0 \qquad R_t < r \qquad t = 1, \cdots, T$$

Let $\mathbf{Z}_t$ be a d-dimensional vector of covariates. Recalling (1), we set

$$P_t(\boldsymbol{\beta}) = P_{\boldsymbol{\beta}}(X_t = 1 | H_{t-1}) = \frac{1}{1 + e^{-\boldsymbol{\beta}'\mathbf{z}_{t-1}}} \quad (4)$$

It should be noted that this model is quite general in the sense that $\mathbf{Z}_{t-1}$ may include past values of X_t and R_t. We include a 1 as the first component of $\mathbf{Z}$ to accommodate for an intercept term in $\boldsymbol{\beta}$. As mentioned earlier, X_t, $\mathbf{Z}_t$ need not be stationary, but $\boldsymbol{\beta}$ is assumed fixed and independent of time.

With this model, the partial likelihood is conveniently given by the product

$$\text{PL}(\boldsymbol{\beta}) = \prod_{t=1}^{T} [p_t(\boldsymbol{\beta})]^{X_t}[1 - p_t(\boldsymbol{\beta})]^{1 - X_t}$$

$$= \prod_{t=1}^{T} \left[\frac{p_t(\boldsymbol{\beta})}{1 - p_t(\boldsymbol{\beta})}\right]^{X_t} [1 - p_t(\boldsymbol{\beta})] \quad (5)$$

But

$$\frac{P_t(\boldsymbol{\beta})}{1 - p_t(\boldsymbol{\beta})} = e^{\boldsymbol{\beta}'\mathbf{Z}_{t-1}}$$

and

$$1 - p_t(\boldsymbol{\beta}) = \frac{1}{1 + e^{-\boldsymbol{\beta}'\mathbf{z}_{t-1}}}$$

so that (5) can be rewritten as

$$\text{PL}(\boldsymbol{\beta}) = \prod_{t=1}^{T} [e^{\boldsymbol{\beta}'\mathbf{z}_{t-1}}]^{X_t} \frac{1}{1 + e^{\boldsymbol{\beta}'\mathbf{z}_{t-1}}} \quad (6)$$

To maximize (6) with respect to $\boldsymbol{\beta}$, we turn to the log partial likelihood

$$\log \text{PL}(\boldsymbol{\beta}) = \sum_{t=1}^{T} [X_t \mathbf{Z}^*_{t-1}\boldsymbol{\beta} - \log (1 + e^{\mathbf{Z}'_{t-1}\boldsymbol{\beta}})] \quad (7)$$

Setting the partial derivative of (7) equal to zero, we obtain

$$\sum_{t=1}^{T} X_t Z_{t-1} = \sum_{t=1}^{T} \left[\frac{e^{\boldsymbol{\beta}'\mathbf{Z}_{t-1}}}{1 + e^{\boldsymbol{\beta}'\mathbf{Z}_{t-1}}}\right] \mathbf{Z}_{t-1}$$

$$= \sum_{t=1}^{T} p_t(\boldsymbol{\beta})\mathbf{Z}_{t-1} \quad (8)$$

If we define

$$\mathbf{Z}' \equiv (\mathbf{Z}_0, \cdots, \mathbf{Z}_{T-1}) \qquad \mathbf{X} \equiv (X_1, \cdots, X_T)$$

$$\mathbf{p}'(\boldsymbol{\beta}) \equiv [p_t(\boldsymbol{\beta}), \cdots, p_T(\boldsymbol{\beta})]$$

then (8) can be rewritten more compactly as

$$\mathbf{Z}'\mathbf{X} = \mathbf{Z}'\mathbf{p}(\boldsymbol{\beta}) \quad (9)$$

The solution of (8) or (9) with respect to $\boldsymbol{\beta}$ is the desired MPLE $\hat{\boldsymbol{\beta}}$. Under some regularity conditions, $\hat{\boldsymbol{\beta}}$ is both consistent and asymptotically normally distributed.

2.3. *Consistency of Maximum Partial Likelihood Estimates*

The large sample theory of MPLE has been developed by several authors including *Andersen and Gill* [1982], *Wong* [1986], and E. Slud (unpublished manuscript, 1990). The conditions for consistency and asymptotic normality appropriate for the present case of logistic regression are given by *Slud and Kedem* [1988]. The proof depends on the fact that

$$S_t(\boldsymbol{\beta}) \equiv \frac{\partial}{\partial \beta} \log \text{PL}(\boldsymbol{\beta}) = \sum_{s=1}^{t} \mathbf{Z}_{s-1}[X_s - p_s(\boldsymbol{\beta})]$$

$t = 1, \cdots, T$, is a Martingale with respect to the histories generated by $\{\mathbf{Z}_t, X_t\}$. Thus one can apply the central limit theorem for Martingales to $S_T(\boldsymbol{\beta}_0)/\sqrt{T}$. For this purpose we must impose a stability criterion on the Hessian matrix for $-\log$ PL($\boldsymbol{\beta}$) given by

$$I(\boldsymbol{\beta}) \equiv \frac{\partial^2}{\partial \beta \, \partial \beta'} \log \text{PL}(\boldsymbol{\beta}) = \sum_{t=1}^{T} \mathbf{Z}_t \mathbf{Z}^*_t p_t(\boldsymbol{\beta})[1 - p_t(\boldsymbol{\beta})]$$

It should be noted that $I(\boldsymbol{\beta})$ is also the cumulative conditional covariance matrix for $S_t(\boldsymbol{\beta})$. Assuming that

$$I(\nu)/T \rightarrow \Lambda(\boldsymbol{\beta}) \qquad T \rightarrow \infty$$

(in probability). We can show that as $T \rightarrow \infty$,

$$\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \rightarrow N[\mathbf{0}, \Lambda^{-1}(\boldsymbol{\beta})]$$

(in distribution)

3. Simulation of SMMR Brightness Temperatures

In this section, the simulation of SMMR data from GATE radar rain rate data is described. SMMR was flown on board Nimbus 7 from 1978 to 1987. There are five channels in SMMR: 6, 10, 18, 21, and 37 GHz with both horizontal and vertical polarization (except at 21 GHz), thus providing multiple covariate measurements. The size of the FOV of the highest frequency (highest resolution) at 37 GHz is about 28 km on a side, and the resolution at 18 GHz is about 56 km on a side. The high-resolution FOVs at 37 GHz can be degraded to the low-resolution FOVs at 18 GHz, and hence a nominal resolution of 56 km on a side is used in the simulation. The response at the 6 GHz to rain rate is small and fairly linear. It is most sensitive to variations in sea surface temperature. Simulated brightness temperatures at 37 and 18 GHz are used, since these channels are sensitive to rainfall variations. The 6-GHz channel is included to test its use as a background for sea surface temperature correction.

During the GATE (conducted in the summer of 1974), radar rain rates are collected by radar on board research vessels. The data have been binned into 4×4 km^2 pixels at a temporal resolution of 15 min [*Patterson et al.*, 1979]. GATE is conducted in three roughly triweekly periods, each termed a phase. Data from phases I and II, consisting of 1716 and 1512 15-min snapshots, respectively, are used in our study. To simulate the SMMR data, the 4-km rain rate data are first transformed into microwave brightness temperature through a rain rate/temperature (*R-T*) relation described below. The simulated SMMR brightness temperatures are obtained by averaging over 14×14 (or 196) GATE pixels, thus giving the brightness temperature of a 56×56 km^2 SMMR FOVs.

The relation between rain rate and microwave brightness temperatures at various frequencies can be calculated using a model of the atmosphere. We used the model results of *Wilheit et al.* [1977] for our *R-T* relation. For the rain sensitive channels of 18 and 37 GHz at the vertical (V) and horizontal (H) polarizations, the *R-T* relation is fitted to the form

$$T(R) = a + b \exp(-cR) + dR$$

where T is microwave brightness temperature in degrees Kelvin at the various frequencies and polarizations, R is rain rate in mm/h, and a, b, c, and d are coefficients obtained by curve fitting. The term $a + b$ is the background (no rain) microwave brightness temperature over the FOV. The exponential term represents radiation due to emission, and the linear term represents the effect of scattering. The response of the microwave temperature to rain rates at the low frequencies is fairly linear, and the functional form used for the 6-GHz channel is

$$T(R) = \alpha + \beta R + \gamma R^2$$

where α, β, and γ are again coefficients.

The rain rate and temperature values used for the fit are the model results of *Wilheit et al.* [1977] for zero surface wind speed and a rain column of 4 km. Table 1 gives the coefficients for the 37-, 18-, and 6-GHz vertical (V) and horizontal (H) channels. Figure 1 shows the *R-T* relation for the various channels.

TABLE 1. Coefficients of the Rain/Rate Temperature Relation

Channel	a, K	b, K	c, (mm/h)$^{-1}$	d, K (mm/h)$^{-1}$
37 GHz				
V	273.586	−47.286	0.847	−1.280
H	272.716	−94.416	0.749	−1.516
18 GHz				
V	293.107	−94.907	0.159	−0.980
H	306.701	−165.201	0.149	−1.641
	α, K	β, K (mm/h)$^{-1}$		γ, K (mm/h)$^{-2}$
6 GHz				
V	158.5	2.174		−0.005
H	86.5	3.284		−0.005

4. Analyses of Simulated SMMR Data

We apply the logistic regression model and the method of partial likelihood estimates on the simulated SMMR data. As before, R_t denotes the rain rate averaged over an SMMR FOV at time t. The covariates we used are V37, H37, V18, H18, V6, and H6, representing the microwave brightness temperatures of the vertical (V) and horizontal (H) polarizations at 37, 18, and 6 GHz. The simulated SMMR time series contain missing values in time, so that the use of the covariates at time lags is not possible. But it should be emphasized that the theory of partial likelihood is not affected by missing values as long as the histories we observed are nested as in (2).

Various combinations of the covariates have been studied. The quantity

$$Y_t = \boldsymbol{\beta}'\mathbf{z}_{t-1}$$

is known as the logit score. Table 2 shows the logit scores and the corresponding covariates for the different models we studied.

4.1. *Definition of Residuals*

To assess the goodness of fit of the logistic model, we define the following types of residual. The first type is defined by

$$e_1(t) = X_t - p_t(\boldsymbol{\beta}) \qquad t = 1, \cdots, T \tag{10}$$

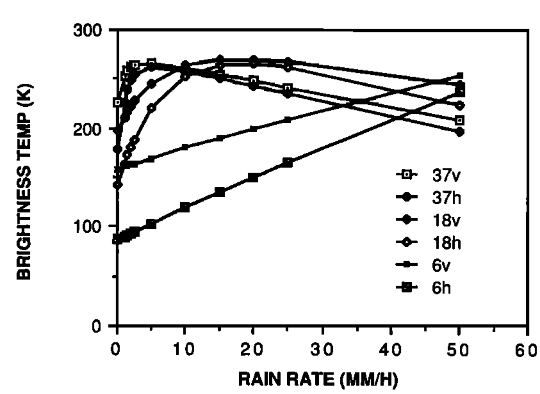

Fig. 1. Graphs of the *R-T* relations used in the simulation of temperature.

TABLE 2. Covariates and the Corresponding Logit Scores

Logit Score	Covariates
Y1	Z1 = (V37, H37)′
Y2	Z2 = (V18, H18)′
Y3	Z3 = (V6, H6)′
Y4	Z4 = (V37, H37, V18, H18)′
Y5	Z5 = (V37, H37, V6, H6)′
Y6	Z6 = (V18, H18, V6. H6)′
Y7	Z7 = (V37, H37, V18, H18, V6, H6)′

where a binary variate is compared with its predicted probability. In the following, the term "residual" always refers to the type 1 residual. If classification has to be performed, a decision has to be made based on the predicted probabilities. Let the classification be defined according to the binary series

$$\chi_t = 1 \qquad P_t(\boldsymbol{\beta}) \geqq 0.5$$

$$\chi_t = 0 \qquad P_t(\boldsymbol{\beta}) < 0.5$$

The type 2 residual is defined as

$$e_2 = X_t - \chi_t \qquad t = 1, \cdots, T \tag{11}$$

This type of residual is binary, since e_2^2 is either zero or 1.

Another objective of this exercise is to compare the goodness of fit of logistic regression with that of simple multiple regression. If $\hat{R}_t$ is the predicted value of R_t from multiple regression, we can define

$$\hat{X}_t = 1 \qquad R_t \geqq r$$

$$\hat{X}_t = 0 \qquad R_t < r$$

and the corresponding type 3 binary residual from prediction using multiple regression

$$e_3(t) = X_t - \hat{X}_t \qquad t = 1, \cdots, T \tag{12}$$

Comparison of the logistic and multiple regression models can be made by comparing the residuals e_2 and e_3.

In all cases, the goodness of fit is judged by the average squared residual

$$1/T \sum_{t=1}^{T} e_j^2(t) \qquad j = 1, 2, 3$$

4.2. *GATE Phase I*

Two FOVs are chosen in the GATE area, and the associated time series of the simulated SMMR microwave brightness temperatures are obtained. Discarding missing values in the time series, there are 1706 time points in each time series, i.e., T = 1706. Both logistic regression and multiple regression have been performed on the time series.

The regression was performed on one time series, while the other time series was used for prediction. Table 3 shows the logit scores, regression coefficients, and log-partial likelihood estimates corresponding to different models defined in Table 2 and for thresholds of 3, 4, and 5 mm/h. The entry of "perfect" in the column of maximum log-partial likelihood means a value very close to zero. The estimated log-partial likelihood, which is a measure of the goodness of fit, shows that the logistic regression model is almost a perfect fit to the data.

The log-partial likelihood is closer to zero for the logit score Y3, which contains the 6-GHz channels, than either Y1 or Y2. This artifact probably arises from the linearity of the R-T relation for the 6-GHz channels. Note also that the regression coefficients for the horizontal and vertical polarizations at the same frequency have opposite signs and a ratio of roughly 1 to 2. *Spencer* [1986] proposed a formula for

TABLE 3. Maximum Partial Likelihood Estimates of $\boldsymbol{\beta}$ Corresponding to Different Thresholds and Different Covariates (T = 1706)

Logit Score	Constant	V37	H37	V18	H18	V6	H6	Log PL($\hat{\beta}$)
				Threshold r = 3 mm/h				
Y1	8105.4	−64.31	36.02					−13.634
Y2	28795.0			−266.03	168.84			−9.08
Y3	22294.0					−868.11	1324.5	−2.77
Y4	26725.0	−2.73	1.39	−244.23	155.55			−9.06
Y5	−7581.9	−12.14	6.54			43.39	25.78	perfect
Y6	−9122.0			−27.46	16.80	90.04	−24.50	perfect
Y7	−43580.0	−3.54	0.10	199.62	−119.90	170.48	−61.23	perfect
				Threshold r = 4 mm/h				
Y1	3900.4	−30.76	17.09					−43.65
Y2	20241.0			−186.00	117.25			−20.53
Y3	6164.7					−1435.5	2535.2	−1.91
Y4	141980.0	−77.65	53.70	−1178.90	703.70			−8.43
Y5	17023.0	−6.566	3.353			−270.86	297.59	perfect
Y6	18259.0			−54.89	33.27	−340.89	481.03	perfect
Y7	57208.0	−99.84	58.96	−218.32	121.22	−330.72	383.60	perfect
				Threshold r = 5 mm/h				
Y1	4115.7	−32.26	17.76					−31.51
Y2	14345.0			−131.45	82.56			−18.65
Y3	−21258.0					85.07	87.19	−3.42
Y4	54764.0	−61.64	38.84	−416.14	245.17			−11.98
Y5	−36576.0	10.31	−5.97			304.44	−150.52	perfect
Y6	−33867.0			12.47	−8.81	278.97	−134.61	perfect
Y7	−28880.0	4.163	−1.68	−103.00	61.66	354.74	−189.28	perfect

TABLE 4. Average Squared Residuals From T = 1706 Observations

Covariates	Genuine Residuals			Prediction Residuals		
	$r = 3$	$r = 4$	$r = 5$	$r = 3$	$r = 4$	$r = 5$
V37 H37	0.00251	0.00870	0.00498	0.00514	0.00389	0.00341
V18 H18	0.00185	0.00395	0.00328	0.00178	0.00252	0.00377
V6 H6	0.00107	0.00058	0.00061*	0.00005*	0.00059	0.00015*
V37 H37 V18 H18	0.00193	0.00499	0.00243	0.00210	0.00274	0.00229
V37 H37 V6 H6	0.00047*	0.00040	0.00087	0.00087	0.00041	0.00121
V18 H18 V6 H6	0.00085	0.00021	0.00084	0.00145	0.00013*	0.00066
V37 H37 V18 H18 V6 H6	0.00364	0.00014*	0.00092	0.00885	0.00043	0.00193

Here, r is in mm/h.
*Minimum value.

estimating oceanic rain rates using the vertical and horizontal channels at 37 GHz. It is interesting to note that the ratio of his coefficients for the horizontal and vertical channels is also about 0.5 (the coefficient b in (1) of *Spencer* [1986]).

Table 4 gives the average squared residuals (type 1) obtained from the maximum log-likelihood estimate of the logistic model on the first time series and the averaged squared prediction residuals (type 2) when the logistic model is applied to the second time series. It can be seen that both the genuine and prediction residuals attain their minimum when the covariates V6 and H6 are included in the logit score. At a threshold level of 5 mm/h, the combination of these two covariates outperforms all other combinations. At a threshold of 4 mm/h, the best model is given by Y6, which also contains V6 and H6 in the combination of covariates.

Does the logistic regression perform better than simple multiple regression in predicting the exceedance of a given threshold? Table 5 compares the squared type 2 and type 3 binary residuals. It can be seen that logistic regression outperforms multiple regression in general. A plausible explanation is that while multiple regression maximizes the variance of the time series, logistic regression maximizes the likelihood function, which is defined relative to an event, such as exceedance of a level. In this respect, it is anticipated that logistic regression will outperform multiple regression in the prediction of exceedance of a given threshold level.

4.3. *Presence of Noise in the Data*

The logistic regression is useful in selecting important covariates for satellite rain rate estimation. The results presented above show that the inclusion of the 6-GHz channel is essential, in the absence of noise. However, the brightness temperatures at these frequencies depend on other meteorological parameters such as cloud water content, water vapor, surface winds, and sea surface temperatures, which are quite variable. To ascertain variabilities in the microwave brightness temperature measurements due to meteorological factors, Gaussian noise with zero means are introduced into the covariates, and the predictions are recalculated using the β coefficients estimated from the "clean" covariates. This also serves as a robustness test, which can be used in assessing the sensitivity of the logistic prediction to errors, both instrumental and meteorological.

The prediction residuals for a "noisy" time series are presented in Table 6. Random noise with standard deviation (s.d.) of 0.1 K is added to the time series. It can be seen that the predictions are still quite good in the presence of a small amount of noise. Table 7 shows the genuine and prediction residuals for s.d. of 1.25 K and 1 K, respectively. As the noise level is increased, prediction based on multiple regression collapses almost completely, while logistic regression

TABLE 5. Averaged Squared Binary Residuals From T = 1706 Observations

Covariates	Logistic Regression			Multiple Regression		
	$r = 3$	$r = 4$	$r = 5$	$r = 3$	$r = 4$	$r = 5$
		Genuine Binary Residuals				
V37 H37	0.00352	0.01465	0.00469	0.02052	0.02052	0.00821
V18 H18	0.00293	0.00528	0.00352	0.02227	0.01583	0.01289
V6 H6	0.00117	0.00059	0.00059	0.00117	0.00059	0.00117
V37 H37 V18 H18	0.00293	0.00528	0.00410	0.00410	0.00469	0.00293
V37 H37 V6 H6	0.00000*	0.00059	0.00117	0.00117	0.00059	0.00117
V18 H18 V6 H6	0.00000*	0.00000*	0.00059	0.00117	0.00410	0.00293
V37 H37 V18 H18 V6 H6	0.00410	0.00000*	0.00000*	0.00352	0.00586	0.00410
		Prediction Binary Residuals				
V37 H37	0.00821	0.00410	0.00352	0.01876	0.01172	0.00879
V18 H18	0.00234	0.00293	0.00528	0.01993	0.01055	0.00996
V6 H6	0.00000*	0.00059	0.00000*	0.00176	0.00059	0.00058
V37 H37 V18 H18	0.00293	0.00352	0.00352	0.00703	0.00117	0.00176
V37 H37 V6 H6	0.00176	0.00059	0.00234	0.00176	0.00059	0.00058
V18 H18 V6 H6	0.00117	0.00000*	0.00059	0.00352	0.00293	0.00117
V37 H37 V18 H18 V6 H6	0.00996	0.00117	0.00234	0.00762	0.00352	0.00117

Here, r is in mm/h.
*Perfect prediction.

TABLE 6. Mean Squared Binary Residuals From 1706 Observations Using Noisy Covariates

	Logistic Regression			Multiple Regression		
Covariates	$r = 3$	$r = 4$	$r = 5$	$r = 3$	$r = 4$	$r = 5$
		Genuine Binary Residuals				
V37 H37	0.02052	0.02521	0.02052	0.06858	0.04220	0.02638
V18 H18	0.14068	0.10727	0.07737	0.29543	0.25381	0.21043
V6 H6	0.01348	0.01289	0.00762	0.03868	0.02931	0.01878
V37 H37 V18 H18	0.12603	0.25264	0.20750	0.32649	0.28722	0.25147
V18 H18 V6 H6	0.00528	0.02286	0.06096	0.03458	0.02521	0.01876
V37 H37 V18 H18 V6	0.02110	0.01583	0.05803	0.02872	0.02227	0.01583
V37 H37 V18 H18 V6 H6	0.10961	0.02286	0.08675	0.01699	0.01934	0.01465
		Prediction Binary Residuals				
V37 H37	0.02110	0.01407	0.01231	0.06389	0.03107	0.01876
V18 H18	0.11547	0.08909	0.06624	0.25674	0.22333	0.18523
V6 H6	0.00996	0.00703	0.00176	0.03634	0.01876	0.01231
V37 H37 V18 H18	0.10258	0.22274	0.18406	0.28195	0.24971	0.22157
V37 H37 V6 H6	0.00762	0.01465	0.04162	0.03048	0.01699	0.01055
V18 H37 V6 H6	0.01641	0.00938	0.03869	0.02169	0.01465	0.00996
V37 H37 V18 H18 V6 H6	0.09379	0.01289	0.05979	0.01817	0.01289	0.00703

Noise is white normal noise with variance 0.01. Here, r is in mm/h.
*Minimum value.

can still give a reasonable estimate for a threshold level of 5 mm/h using the combination of V6 and H6.

How well do the predicted probabilities $p_t(\beta)$ trace the observed binary series X_t? Table 8 shows some typical examples. When the observed rain rate is far from the threshold, the prediction (classification) of X_t by $p_t(\beta)$ is rather straightforward: $p_t(\beta)$ traces X_t very well and is essentially binary itself. In this case, almost perfect prediction can be achieved. If the observed rain rate is close to the threshold level, the prediction is less accurate. Errors in prediction may occur when R_t is very close to the threshold level.

4.4. *GATE Phase II*

Similar analyses were performed on two simulated SMMR time series obtained from phase II of GATE. The average rain rate in GATE phase II is lower than that in phase I. This means that the optimal threshold levels must be lower than those of GATE phase I. Similar results are obtained, and the results are not reproduced in detail here. It can be pointed out that for a threshold level of 1 mm/h the combination of V6 and H6 again showed up as the best model in the logistic predictions. However, when appreciable noise was introduced into the covariates, multiple regression outperformed logistic regression at this threshold level.

5. ANALYSES OF ESMR 5 DATA

Chiu and Kedem [1986] considered the scenario of concomitant rain rate and ESMR 5 measurements. Recently, colocated and coincident ESMR 5 and rain rate measurements during the GATE period have been assembled by *Short* [1988]. Empirical corrections to the ESMR 5 measurements have been made to account for scan angle dependence. The spatial average rain rate over an ESMR 5 FOV

TABLE 7. Mean Squared Binary Residuals From 1706 Observations and Contaminated Covariates

	Logistic Regression			Multiple Regression		
Covariates	$r = 3$	$r = 4$	$r = 5$	$r = 3$	$r = 4$	$r = 5$
		Genuine Binary Residuals (Noise ~ N(0.15625))				
V37 H37	0.35111	0.34467	0.32767	0.40445	0.38218	0.36401
V18 H18	0.45604	0.45545	0.45487	0.49179	0.48652	0.47890
V6 H6	0.33001	0.28488	0.09613*	0.39097	0.38687	0.36284
V37 H37 V18 H18	0.45252	0.48945	0.47655	0.49414	0.48769	0.47948
V37 H37 V6 H6	0.20164	0.36225	0.42497	0.38335	0.37456	0.35346
V18 H37 V6 H6	0.37163	0.31653	0.42087	0.37339	0.35580	0.33646
V37 H37 V18 H18 V6 H6	0.42556	0.35229	0.44314	0.34291	0.32474	0.30246
		Prediction Binary Residuals (Noise ~ N(0.1))				
V37 H37	0.35229	0.33822	0.32591	0.40973	0.39332	0.37456
V18 H18	0.42732	0.42204	0.42145	0.46073	0.45311	0.45252
V6 H6	0.30305	0.24267	0.05451*	0.37456	0.35522	0.33294
V37 H37 V18 H18	0.42204	0.45780	0.45252	0.47245	0.46717	0.46424
V18 H18 V6 H6	0.15182	0.03305	0.40739	0.36694	0.34525	0.32356
V37 H37 V18 H18 V6	0.32005	0.28546	0.39918	0.35346	0.33411	0.30539
V37 H37 V18 H18 V6 H6	0.40621	0.32767	0.41383	0.33177	0.30188	0.27315

Here, r is in mm/h.
*Minimum value.

TABLE 8. Typical Prediction of x_t by $pt(\hat{\beta})$ for Different Combinations of Thresholds and Covariates

Threshold r = 5 mm/h Covariates V6 H6			Threshold r = 5 mm/h Covariates V37 H37 V6 H6			Threshold r = 3 mm/h Covariates V37 H37 V6 H6		
R_t	X_t	$pt(\hat{\beta})$	R_t	X_t	$pt(\hat{\beta})$	R_t	X_t	$pt(\hat{\beta})$
1.855	0	0.000	11.512	1	1.000	3.067	1	0.659
11.754	1	1.000	4.919	0	0.041	2.956	0	0.287
6.672	1	1.000	1.632	0	0.000	3.564	1	0.999
7.337	1	1.000	4.013	0	0.000	3.414	1	0.999
5.319	1	1.000	4.613	0	0.003	3.604	1	0.999
4.392	0	0.000	5.849	1	0.999	3.388	1	0.999
5.210	1	0.999	3.697	0	0.000	2.742	0	0.026
3.742	0	0.000	4.904	0	0.165	1.022	0	0.000
3.128	0	0.000	4.153	0	0.000	2.675	0	0.003
2.576	0	0.000	6.146	1	1.000	2.513	0	0.000
1.592	0	0.000	4.370	0	0.000	2.260	0	0.000
0.905	0	0.000	3.555	0	0.000	2.793	0	0.039
4.992	0	0.233	1.941	0	0.000	5.494	1	1.000
4.607	0	0.000	4.224	0	0.000	5.833	1	1.000
4.611	0	0.000	3.575	0	0.000	3.856	1	1.000

was calculated by weighting the GATE radar measurements (4×4 km^2 pixels) by the antenna pattern of ESMR [*Short*, 1988]. In addition to the ESMR 5 microwave brightness temperature, we include the fractional area with rain rates above 0, 1, 2, 3, 4, and 5 mm/h in the ESMR 5 FOV. These covariates are denoted F0, F1, F2, F3, F4, F5, respectively. Finally, the variance of rain rate within each ESMR 5 FOV, calculated from the 4×4 km^2 pixels, denoted VR, was also included as a covariate. All together, there are 511 FOVs. Admittedly, some of the variables will be difficult if not impossible to obtain from space observations; it is nonetheless instructive to evaluate the logistic regression in the classification scheme. The importance of the covariates can also be assessed.

Table 9 shows the means of and correlations between the covariates. It can be seen that the fractions of the areas within the FOV for different thresholds are highly correlated. RR, the average rain rate over the FOV is best correlated with F5. This is consistent with the findings of *Chiu* [1988*a*, *b*] and *Rosenfeld et al.* [this issue]. It is interesting to note that the high correlation between rain area and areal average rain rate holds even for small areas: in this case the area of a ESMR 5 FOV, which is about 10^3 km^2.

Logistic regression and multiple regression were performed on the ESMR 5 data set. Results from the logistic regression and multiple regression for various combinations of the covariates are presented in Tables 10 and 11, respectively. It can be seen that the brightness temperature is a fairly poor predictor by itself. However, by including other covariates, the regression improved significantly.

Table 12 shows the square binary residuals for the logistic regression and multiple regression. For the same combination of the covariates, the logistic regression models again outperform the corresponding multiple regression models at all threshold levels. Examination of the residuals showed that for rain rates far away from the threshold, both methods perform well. But for rain rates close to the threshold, the performance of logistic regression is still acceptable, while that of multiple regression is poor.

A prediction was performed by partitioning the ESMR 5 data into two sets. The first set, consisting of 300 observations, was used for the regression analysis. The result from this regression analysis was then applied to the remaining 211 observations. The model we used contains covariates T, VR, F0, F1, F2, F3, and F4. The regression coefficients, the maximum log-partial likelihood, and the average square residuals appear in Table 13. The prediction residuals for thresholds of 1, 4, and 6 mm/h are less than 1.5%, meaning that there are only three or fewer misclassifications in the 211 FOVs.

6. Concluding Remarks

A logistic regression model for estimating the exceedance probability of rain rates above certain thresholds, given covariate information, was presented. Logistic regression

TABLE 9. Table of Correlation Coefficients Between Covariates

	RR, mm/h	F0	F1	F2	F3	F4	F5	VR	FOV T, K
Mean	2.821	0.500	0.389	0.320	0.249	0.195	0.164	28.56	185.85
RR		0.702	0.783	0.813	0.868	0.909	0.929	0.792	0.856
F0			0.936	0.883	0.828	0.772	0.724	0.453	0.770
F1				0.982	0.937	0.877	0.823	0.498	0.838
F2					0.973	0.919	0.866	0.514	0.853
F3						0.978	0.940	0.566	0.879
F4							0.986	0.613	0.884
F5								0.641	0.878
VR									0.596

TABLE 10. Maximum Partial Likelihood Estimates for β Corresponding to Different Thresholds From the Real ESMR Data [*Short*, 1988]

Constant	T	VR	F0	F1	F2	F3	F4	F5	R^2
Threshold r = 1 mm/h									
−53.498	0.300								−166.540
−24.853	0.018	1.261		58.846					−15.823
−61.222	0.258	0.830			69.628				−20.991
−28.294	0.064	0.603	1.962	35.434	7.123	−30.170	91.843		perfect
Threshold r = 2 mm/h									
−39.415	0.213								−175.530
−17.988	−0.045	0.297	15.699	4.221	13.168	−3.285	52.549		−21.854
−17.403	−0.053	0.308	16.231	3.581	14.533	6.644	15.396	30.284	−21.307
Threshold r = 3 mm/h									
−35.368	0.185								−158.000
−40.313	0.022	0.281	1.338	17.966	−0.715	14.775	57.768		−14.837
−201.270	−0.601	2.163	68.338	96.116	72.313	419.420	−938.830	1115.300	−4.906
Threshold r = 4 mm/h									
−32.336	0.164								−135.37
−39.516	0.116	0.140					35.292		−23.716
−40.028	0.071	0.202	12.682	0.524	1.402	0.737	32.071		−18.147
−44.176	0.064	0.215	10.976	−2.496	6.923	16.072	−9.850	35.223	−15.601
Threshold r = 5 mm/h									
−35.327	0.174								−94.961
−35.035	0.113	0.065						27.345	−27.587
−36.185	0.079	0.098	−2.778	20.278	−7.771	−13.682	37.273		−21.794
−36.920	0.076	0.102	2.243	19.542	−8.402	−7.392	23.760	12.236	−21.026
Threshold r = 6 mm/h									
−35.446	0.171								−77.554
−31.206	0.047	0.091	9.684	−9.605	17.628	−17.609	26.420		−17.885
−30.588	0.033	0.095	9.127	−3.132	13.909	−14.489	8.694	17.189	−16.803

By including more covariates in addition to T, the partial likelihood increases significantly.

performs remarkably well even when the observed rain rate is close to the threshold level. Comparison with multiple regression shows that logistic regression is superior in predicting the occurrence of exceedances. Application of these regression techniques to simulated SMMR and actual ESMR 5 data during the GATE period indicates that accurate covariate information consisting of microwave brightness temperatures at various frequencies can lead to precise prediction of exceedance.

The logistic regression model is also applicable for noisy covariates, provided that the noise level is not very high (s.d. = 0.1 K). Fluctuations in meteorological parameters are of the order of a few degrees Kelvin. Hence variabilities of meteorological parameters must be accounted for if the logistic regression is to be applied to estimate rain rate from space.

APPENDIX: A MAXIMUM ENTROPY DERIVATION OF LOGISTIC REGRESSION

The general logistic model which we apply in this paper expresses the dependence of a probability of a binary event on several covariates, or independent variables. The logistic model can be derived from the principle of maximum entropy. The entropy of a distribution measures the degree of uncertainty exhibited by a distribution, and the maximum entropy principle calls for the maximization of the entropy, subject to constraints that represent existing knowledge.

Let R stand for rain rate. Consider the events $A_0 = \{R > r_1\}$, $A_1 = \{r < R \le r_1\}$, $A_2 = \{R \le r\}$ with probabilities P_0, P_1, P_2:

$$P(R > r_1) = P_0 \qquad P(r < R \le r) = p_1$$

$$P(R \le r) = p_2 \qquad p_0 + p_1 + p_2 = 1$$

Call the event of exceedance $\{R > r\}$ a success. Together with these events, we define the states E_0, E_1, E_2, by

$$E_0 \equiv \alpha_0^{(0)} + \alpha_1^{(0)} z_1 + \cdots + \alpha_k^{(0)} z_k$$

$$E_1 \equiv \alpha_0^{(1)} + \alpha_1^{(1)} z_1 + \cdots + \alpha_k^{(1)} z_k$$

$$E_2 \equiv \alpha_0^{(2)} + \alpha_1^{(2)} z_1 + \cdots + \alpha_k^{(2)} z_k$$

TABLE 11. Estimated Coefficients From Multiple Regression of Rain Rate on the Corresponding Covariates

Constant	T	VR	F0	F1	F2	F3	F4	F5	R^2
−34.014	0.198								0.734
−6.187	0.036	0.025					8.530		0.919
−6.347	0.037	0.024	−0.014	0.110	2.840	−9.100	15.300		0.924
−3.800	0.022	0.022	0.100	−0.330	3.300	−1.200	−8.560	18.100	0.937

R^2 increases sharply with the inclusion of more covariates in addition to T.

TABLE 12. Averaged Squared Binary Residuals From 511 Observations

Covariates	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$	$r = 6$
		Genuine Binary Residuals From Logistic Regression				
T	0.14481	0.51656	0.16243	0.12133	0.08219	0.08415
T, VR, F1	0.01370					
T, VR, F2	0.01174					
T, VR, F4				0.02153		
T, HR, F5					0.01957*	
T, VR, F0, F1, F2, F3, F4	0.00000*	0.02153	0.01174*	0.00978	0.01566	0.01761
T, VR, F0, F1, F3, F4, F5		0.01370*	0.08415	0.00783*	0.02544	0.01370*
		Genuine Binary Residuals From Multiple Regression				
T	0.5460	0.14481	0.14873	0.12524	0.09198	0.082192
T, VR, F4	0.3718	0.05284	0.03131	0.03718	0.04305	0.02544
T, VR, F0, F1, F2, F3, F4	0.03522	0.06067	0.04305	0.03914	0.03718	0.02153
T, VR, F0, F1, F2, F3, F4, F5	0.03131	0.04892	0.03914	0.04892	0.03327	0.02153

Threshold values are in mm/h.
*Minimum value for the level. There is always a logistic regression model that outperforms any regression model.

where $z_1, \cdots, z_k$ are covariates. We assume that when the system is in state E_i, then A_i occurs. In other words, we think of E_i as the state corresponding to the event A_i. In this way, we relate covariates to events. We now proceed to relate the covariates to probabilities.

Define the average state $\bar{E}$ by

$$\bar{E} = p_0 E_0 + p_1 E_1 + p_2 E_2 \tag{A1}$$

and note that

$$p_0 + p_1 + p_2 = 1 \tag{A2}$$

These two equations define constraints on p_0, p_1, p_2. Thus the entropy S of p_0, p_1, p_2 subject to these constraints is given by

$$S = -K \sum_1^3 p_i \log p_i + \lambda_0 \left(1 - \sum_1^3 p_i \right) + \lambda_1 \left(\bar{E} - \sum_1^3 p_i E_i \right)$$

where K stands for the appropriate unit of measurement. If we let

$$\gamma_0 = 1 + \frac{\lambda_0}{K} \qquad \gamma_1 = 1 + \frac{\lambda_1}{K}$$

then p_0, p_1, p_2 that maximize S subject to $C1$, $C2$ are given by the Gibbsian form

$$p_i = \exp(-\gamma_1 E_i) \Big/ \sum_{j=i}^{3} \exp(-\gamma_1 E_j) \qquad i = 0, 1, 2$$

See *Jaynes* [1985] for a general formulation of the maximum entropy principle that leads to more general Gibbsian forms.

Now, think of r_1 as a very high level, so that p_0 is small, and note that $p_0 \to 0$ is equivalent to $\gamma_1 E_0 \to \infty$. When p_0 approaches 0, p_1 has the logistic form. To see this, we define

$$\beta_0 = \gamma_1(\alpha_0^{(2)} - \alpha_0^{(1)})$$
$$\beta_1 = \gamma_1(\alpha_1^{(2)} - \alpha_1^{(1)})$$
$$\vdots$$
$$\beta_k = \gamma_1(\alpha_k^{(2)} - \alpha_k^{(1)})$$

On noting that

$$E_2 - E_1 = (\alpha_0^{(2)} - \alpha_0^{(1)}) + \cdots + (\alpha_k^{(2)} - \alpha_k^{(1)}) z_k$$

we obtain the desired approximation for small p_0

TABLE 13. Partial Likelihood Estimates and Averaged Squared Binary Residuals Obtained From the ESMR 5 Data

Covariates	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$	$r = 6$
Constant	−19.755	−7.224	−387.000	−61.181	−40.593	−28.628
T	0.035	−0.097	0.450	0.150	0.106	0.025
VR	0.414	0.263	2.319	0.211	0.101	0.094
F0	7.624	17.045	−143.140	14.464	−2.483	10.694
F1	21.613	−3.769	−14.123	8.711	41.002	−7.540
F2	4.165	21.549	284.700	−7.955	−1.248	12.268
F3	−28.739	−4.187	−16.497	8.148	−9.358	−9.299
F4	82.853	41.299	982.150	26.033	28.797	21.777
log PL($\hat{\beta}$)	perfect	−11.511	perfect	−11.160	−16.978	−15.657
Averaged squared residuals	0.00333	0.01000	0.0000	0.01333	0.02333	0.02333
Averaged squared prediction residuals	0.01422	0.03317	0.03792	0.01422	0.01422	0.00945

Threshold levels are in mm/h.

$$p_1 \simeq \frac{1}{1 + e^{-(\beta_0 + \beta_1 Z_1 + \cdots + \beta_k Z_k)}}$$

or

$$P(R > r) \simeq \frac{1}{1 + e^{-\boldsymbol{\beta}'\mathbf{z}}} \qquad (1')$$

which is the logistic form of the probability of success. We thus arrive at an attractive model for a probability of success, taking into account covariate information. This logistic form is the building block of our partial likelihood.

Acknowledgments. The authors are grateful to David Short of NASA/Goddard Space Flight Center for providing the ESMR 5 data. This research is supported by the National Aeronautics and Space Administration through contract NAS5-30083.

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(Received November 14, 1988;
revised June 26, 1989;
accepted June 28, 1989.)